

LATE-TIME COSMOLOGICAL PHASE TRANSITIONS *

DAVID N. SCHRAMM

The University of Chicago

5640 S. Ellis Avenue, Chicago, IL 60637

and

NASA/Fermilab Astrophysics Center

Box 500, Batavia, IL 60510-0500

ABSTRACT

It is shown that the potential galaxy formation and large-scale structure problems of (1) objects existing at high redshifts ($Z \gtrsim 5$), (2) structures existing on scales of $100Mpc$ as well as velocity flows on such scales, and (3) minimal microwave anisotropies $\frac{\Delta T}{T} \lesssim 10^{-5}$ can be solved if the seeds needed to generate structure form in a vacuum phase transition after decoupling. It is argued that the basic physics of such a phase transition is no more exotic than that utilized in the more traditional GUT scale phase transitions, and that, just as in the GUT case, significant random gaussian fluctuations and/or topological defects can form. Scale lengths of $\sim 100Mpc$ for large-scale structure as well as $\sim 1Mpc$ for galaxy formation occur naturally. Possible support for new physics that might be associated with such a late-time transition comes from the preliminary results of the SAGE solar neutrino experiment, implying neutrino flavor mixing with values similar to those required for a late-time transition. It is also noted that a see-saw model for the neutrino masses might also imply a tau neutrino mass that is an ideal hot dark matter candidate. However, in general either hot or cold dark matter can be consistent with a late-time transition.

* Prepared for Proceedings of IUPAP Conference on "Primordial Nucleosynthesis and Evolution of Early Universe", Tokyo, September 1990



Introduction

The purpose of this paper is to describe the current situation regarding late-time cosmological phase transitions as mechanisms for generating structure in the Universe[1]. This subject has received a tremendous boost by the combination of a variety of preliminary observations regarding large-scale structure and galaxy formation coupled with recent hints that the solar neutrino experiment may require new neutrino physics involving small masses and flavor changing with energy scales involved being appropriate to a late-time phase transition.

In this paper we will discuss plausible late-time phase transitions (LTPT) and compare their plausibility with other mechanisms for generating structure in the Universe. In particular, we will note that the physics of late-time transitions is really no different from the basic physics required for GUT scale phase transitions used in more traditional models. We will also note that the recent reports from the Soviet-American Gallium Experiment (SAGE) seem to imply that neutrino physics involves masses and flavor mixings that might be appropriate to a late-time transition. We will then discuss the range of possible structures that might be generated in an LTPT, noting that they can yield multiple great walls and velocity flows, objects at high redshift, and a variety of intricate and unusual patterns of the type that are beginning to be seen in redshift surveys. In particular, we will note that LTPT can generate small spherical objects such as "bags" or "balls" of wall or textures which can serve as seeds in hot dark matter models for galaxy formation and can possibly explain the seed spacings necessary for Vornoi tessalation models[2,3] of the Broadhurst *et al.*[4,5] data. We will discuss how LTPT can give a combination of both random gaussian fluctuations as well as topological defects in this manner similar to any other phase transition in the early Universe. We will also note that because they form after recombination, many of the topological defects do not have the same consequences as those that formed before recombination. In particular, domain wall models can be made to work if the walls decay, or come from multiple minima, or have friction so that they move slowly, or if the walls split off into bags or balls of wall. All such models can yield interesting and exciting structures. Only in the case where infinite, stable walls dominate does one run into the one-wall domination problems of Ref. [6,7,8].

We will also discuss the resulting microwave fluctuations from LTPT. We will note that for a given size structure, if it is generated by an LTPT, one will obtain the minimal $\frac{\Delta T}{T}$ for that structure. Since an LTPT does not require fluctuations on the surface of last scattering, all induced microwave anisotropies are due to propagations through transparent medium effects.

This paper will conclude with a discussion of future obserations and how they will help verify or rule out LTPT. The paper will also outline future calculations that will be important in establishing or eliminating LTPT as a model for large-scale structure information.

Structure Formation

Before specifically going into late-time phase transitions, let us review the basic framework of structure formation in the Universe. In particular, let us note that structure formation requires that density fluctuations grow. In order for this to occur, $\rho_{m(atter)}$ must be greater than $\rho_{r(adiation)}$. If we define T_{eq} as the temperature where $\rho_m = \rho_r$, then for an $\Omega = 1$ universe with h_0 (the Hubble constant in units of $100km/sec/Mpc$) equal to 0.5,

equality is approximately 10^4 times the present temperature T_0 . The horizon mass at T_{eq} is $\sim 5 \times 10^{16} M_\odot$ which gives a present comoving scale of $\sim 60 Mpc$. The recombination epoch T_{rec} for an $\Omega = 1$ universe occurs slightly after matter domination. At $T \sim 1000T_0$ baryon fluctuations begin to grow after recombination and the horizon mass at recombination is about $10^{18} M_\odot$ with a comoving scale of $200 Mpc$. We also know that the fluctuations in the microwave background temperature at the time of recombination are less than a few parts in 10^5 [9]. Thus, in traditional models with primordial fluctuations existing prior to matter domination, growth begins at matter domination with the limits from $\frac{\delta T}{T}$ forcing $\frac{\delta \rho}{\rho}$ to be less than the order of 10^{-4} since

$$\frac{\delta \rho_m}{\rho} \lesssim 3 \frac{\delta T}{T} \lesssim 10^{-4}.$$

Since small fluctuation $\delta \rho$ grows linearly with $1 + z$, this would mean that fluctuations could reach the order of unity only at the present epoch. Non-linear growth, and thus true structure formation, does not begin until $\frac{\delta \rho}{\rho}$ has reached unity (see Figure 1). Thus, in the standard model, the existence of objects at $z > 1$ (see for example Gunn, Schneider, and Schmidt [10]) requires that there be fluctuations far larger than the average in order that these objects currently exist. As Efstathiou and Rees [11] point out, the gaussian fluctuation model for primordial fluctuations would not allow a large number of quasar-like objects to form at $z \gtrsim 5$.

All models for structure formation require at least two basic ingredients for that structure:

- (1) the matter,
- (2) the seeds.

In traditional models, the seeds are random fluctuations in the density field generated at the end of the GUT phase transition, presumably accompanying inflation [12,13].

The matter in any model of galaxy formation with $\Omega = 1$ consists of normal baryonic matter with Ω the order of 0.06 and some non-baryonic matter, either hot or cold, with Ω the order of 0.94. [14]. This is summarized in Table 1.

As was emphasized in reference [14], the robustness of the nucleosynthetic constraint telling us that $\Omega_{b(\text{aryon})}$ is about 0.06 seems very solid. Since Ω associated with shining matter is less than 0.01, this tells us that the bulk of the baryons in the Universe are dark. Whether they are in condensed objects that would be in the halos of galaxies, such as brown dwarfs, jupiters or black holes, or whether they are in the form of some hot intergalactic gas at a temperature high enough to avoid the Gunn-Peterson Test, but low enough to avoid significant x-ray emission, or in the form of stillborn galaxies remains to be determined. In any case, it does seem clear that dark baryons must exist somewhere.

As to the non-baryonic dark matter the question remains as to whether it is in the form of matter that is slow moving at the time of galaxy formation, which has been dubbed "cold dark matter," or whether it is fast moving at the time of galaxy formation and dubbed "hot dark matter."

The seeds which clump the matter to form objects may be divided into two broad categories (see Table 2) which can further be subdivided. The two broad categories would be (1) random gaussian seeds, presumably induced by quantum fluctuations at the end of a phase transition, and (2) topological defects produced in a vacuum phase transition. For the random gaussian seeds, the traditional assumption has been that the phase transition

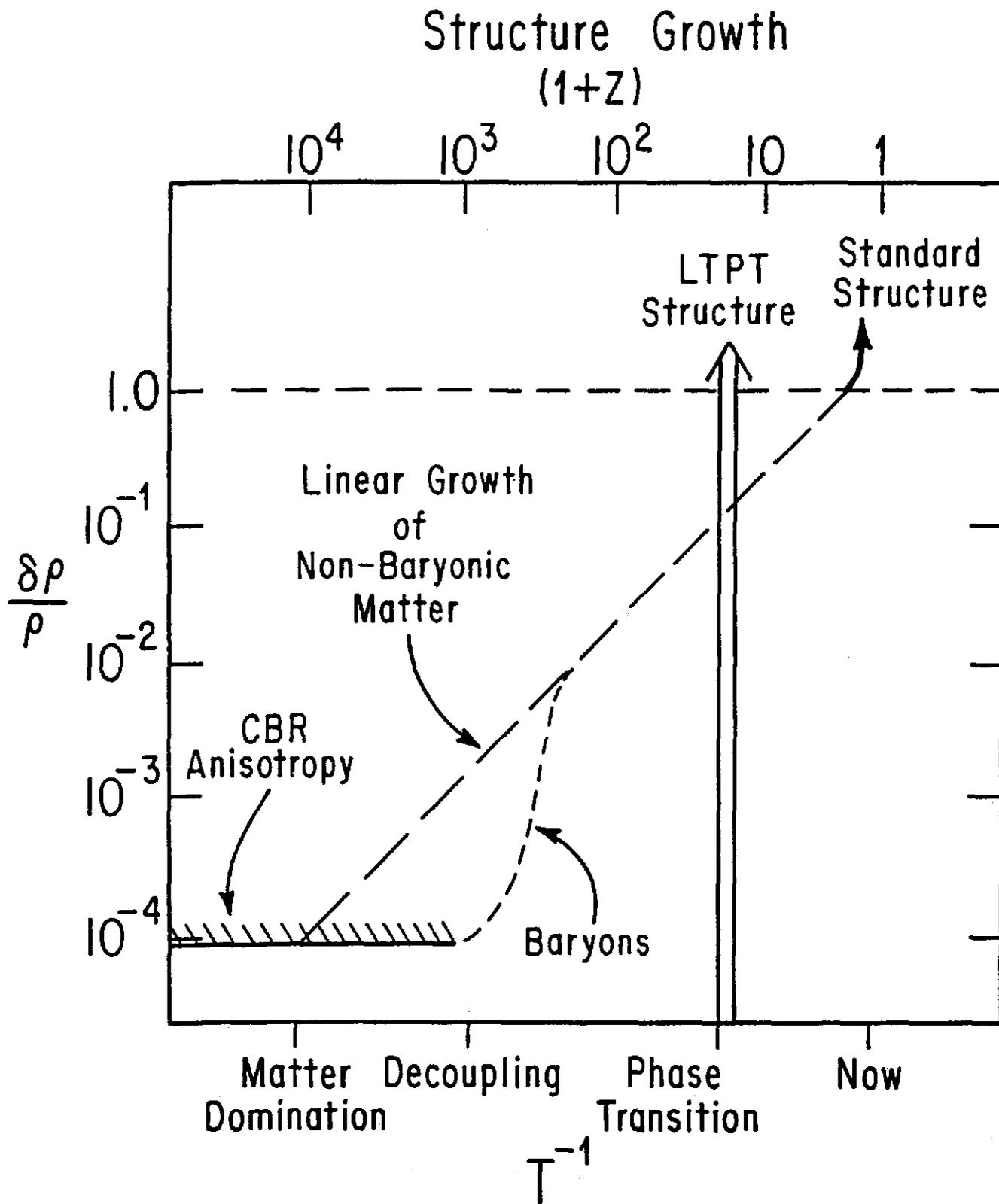


Figure 1. The growth of density fluctuations with the expansion of the universe. Note that LTPT can yield non-linear growth and thus structure at epochs much earlier than standard primordial models.

Table 1
MATTER

Baryonic $\Omega_b \sim 0.06$

VISIBLE $\Omega_{vis} \lesssim 0.01$

DARK

Halo

Jupiters

Brown Dwarfs

Stellar Black Holes

Intergalactic

Hot gas at $T \sim 10^5 K$

Stillborn Galaxies

Non Baryonic $\Omega_{nb} \sim 0.94$

HOT

$m_{\nu_\tau} \sim 25 eV$

COLD

Wimps/Inos $\sim 100 Gev$

Axions $\sim 10^{-5} eV$

Planetary Mass Black Holes

Table 2

SEEDS

- I RANDOM GAUSSIAN (Quantum)
 - A. End of Inflation
 - B. LTPT

- II TOPOLOGICAL DEFECTS
 - A. GUT
 - B. LTPT

is the one associated with inflation[12,13]. However, it has been shown that similar kinds of fluctuations can also be generated in late-time phase transitions[15,16]. Similarly, for the topological defects, they could be formed either at the end of a GUT phase transition ($\sim 10^{15} GeV$) or in some late-time transition[1,17,18]. In some sense this current division of random versus topological replaces the old division of adiabatic versus isothermal (or isocurvature). In fact, the current “random gaussian” are indeed “adiabatic” and the topological are isothermal and isocurvature. However, the latter have the new added feature of also being non-gaussian.

Let’s note that all models for galaxy formation require new fundamental physics beyond

$$SU_3 \times SU_2 \times U_1.$$

In particular, all non-baryonic dark matter, whether hot or cold, requires new physics, and similarly, all seeds, whether GUT scale or late-time and whether random gaussian or whether topological, require vacuum phase transitions. No model exists that does not invoke new physics. In fact, the existence of structure in the Universe is one of the most important clues to the existence of physics beyond the standard model.

We should also note that not all combinations of seeds and matter are possible. For example, if one uses random gaussian seeds, then the non-baryonic matter must be cold, whereas if one uses topological seeds, the non-baryonic matter can be either hot or cold. One should also note that baryonic halos would require hot dark matter and hence topological seeds. Thus, searches for the dark baryons will also help constrain the non-baryonic candidates.

All seed models require some form of vacuum phase transition. Thus, let us explore what possible phase transitions might occur (see Table 3). It should be noted in looking at Table 3 that of the three general classifications of cosmological phase transitions—the early, intermediate and late—the only ones that we absolutely know must have occurred are in the intermediate category when there is a horizon problem, namely that the horizon at the time of that transition is too small to generate galactic sized structure, and yet, the transition is not accompanied by significant inflation. The traditional early transitions have been used in the past because, while their horizon is small, inflation can amplify the effects to large scales. The other option, which we are advocating in this paper, is that of a late-time transition, where the universe waits until the horizon is sufficiently large that the physics of the phase transition directly yields the structures without having to use inflation to avoid the horizon problem.

Potential Observations to be Explained

In the last couple of years there have been a number of observations affecting galaxy formation and large-scale structure that have been a potential problem for traditional models which invoked early random gaussian fluctuations. However, because each of these observations is new and has not stood the test of time, in this paper we refer to these as potential observations. In particular, many of the advocates of gaussian fluctuations and cold dark matter have tried to argue that these observations are statistical flukes that have yet to be established. Obviously, if these potential observations continue to hold up and are verified and are shown to be ubiquitous rather than statistical rareties, then the traditional models are in serious trouble. Table 4 summarizes these potential observations. Perhaps the most potentially damning would be observations of microwave anisotropies $\frac{\Delta T}{T}$

Table 3

VACUUM PHASE TRANSITIONS

EARLY	(Small horizon but inflation) $\sim 10^{19} GeV$ - T.O.E. $\sim 10^{16} GeV$ - GUT
INTERMEDIATE	(Known to occur but horizon problem) $\sim 10^2 GeV$ - Electroweak $\sim 1 GeV$ - QCD
LATE	(Horizon large) $\sim 10^{-2} eV$ - Family symmetries, etc.

Table 4

POTENTIAL OBSERVATIONS

1. $\frac{\delta T}{T} \lesssim 10^{-5}$
2. Structures $\gtrsim 100 Mpc$
3. Large coherent velocity flows
4. Objects existing at $z \gtrsim 5$
5. Large cluster - cluster correlations

at levels significantly below 10^{-5} . However, at the present time, observations of small scale anisotropy are at the level of a couple times 10^{-5} . Observations on angular scales of degrees or more are also approaching a few 10^{-5} . As this paper is being written, the measurements have not yet reached the point of ruling out the model of random fluctuations. However, as noted by Smoot[9], within the not too distant future, COBE may be able to achieve limits as low as 3×10^{-6} on scales of a few degrees and larger, and antarctic studies may also push to similar levels on somewhat smaller scales, as might the balloon studies of Meyer at MIT.

The next observation that can be a potential problem for traditional models is the existence of structures with scales greater than the order of $100Mpc$. In particular, the great wall observed by Geller and Huchra[19] shows that there is at least one such wall in the Universe. The observations of Broadhurst *et al.*[4,5] show evidence for a multiplicity of such great walls with the characteristic spacing comparable to the size of the Geller-Huchra wall itself. While much debate has been made about whether or not the multiple walls of Broadhurst *et al.* are periodic or quasi-periodic, it does seem clear from their observations, as well as the work reported by Szalay[20], that there is significant structure in the Universe on scales of $\sim 100Mpc$. This is thoroughly supported by the large coherent velocity flows where the Seven Samurai[21] and others have found evidence for the existence of an object they call the "Great Attractor" towards which the Virgo cluster and the Hydro-Centaurus cluster all seem to be flowing with a velocity $\sim 600km/sec$. This again seems to indicate evidence of structures on the scales of at least $60Mpc$.

Perhaps most constraining of the traditional astronomical measurements is the existence of objects at very large redshifts. In particular, Gunn, Schneider, and Schmidt[10] have found a quasar with a redshift of 4.73. As Efstathiou and Rees[11] have noted, if such objects are ubiquitous, this would be fatal for primordial gaussian fluctuation models. Similarly, if one ever finds an a quasar-type object at much larger redshifts, that would also be fatal.

Another potentially fatal observation for gaussian fluctuation models comes from the work of Bahcall and Soneira[22], and Klypin and Khlopov[23] where they find that clusters of galaxies seem to be more strongly correlated with each other than galaxies are correlated with each other. While Primack and Dekel[24] have warned of the dangers of projection effects on such observations, it seems difficult to understand how projection effects would give the fractal-like behavior[25]. Furthermore, the southern hemisphere work of Huchra[26] also seems to support high cluster correlations. Most recently van den Bergh and West[27] have also found similar correlations for the CD galaxies observed at cluster centers. The CD's should not have the projection effect problems because redshifts are known. Even Primack and Dekel now acknowledge that there seems to be some excess in cluster correlations. If such large correlations turn out to be real, they too cannot be easily explained in the gaussian model, and, as Szalay and Schramm[25] note, they seem to be best fit by some sort of fractal-like pattern, as one might get from topological defects induced by a phase transition.

Late-Time Transitions

By late-time transition we will mean any non-linear growth occurring shortly after recombination. As mentioned above, such non-linear growth can be related either to a gaussian pattern or to a topological pattern such as walls, strings or textures. It is also possible that some normal random gaussian pattern from the very early universe could be triggered to

undergo non-linear growth by some sort of phase transition or related phenomenon occurring after recombination. An example of this latter case would be the neutrino flypaper model of Fuller and Schramm[28].

In general we will see that these late-time transitions can give the smallest possible $\frac{\Delta T}{T}$ for a given size structure. They can produce non-gaussian structural patterns, fractal-like with large velocity flows. It might be noted that the co-moving horizon at the time of the transition is not too different than the scale associated with the largest structures observed. No model of primordial fluctuations naturally imbeds this horizon scale onto the structural pattern. If some non-linear growth is associated with the patterns, the horizon scale can be imposed on the structure.

Another very dramatic advantage of late-time transitions, illustrated in Fig. 1, is that it can produce structure with $\frac{\delta\rho}{\rho} \gtrsim 1$ at $z \gtrsim 10$. Thus, one could have significant structure and a significant number of objects at high redshift, which is a problem in any normal model with the seeds forming prior to recombination.

Let us now explore the possible physics that might give rise to a late-time transition, that is, a transition with a critical temperature between $0.001eV$ and $1eV$. It might be noted that in some sense it is a “hierarchy” rather than a “fine-tuning” problem to obtain a transition in this temperature range. We are trying to find a small mass scale somewhat analogous to how one would like to find the mass scale of the electron, or, for that matter, the Z^0 boson, when the natural mass scales to the problem are closer to $10^{19}GeV$, as in superstring models, or to 0. The hierarchy problem of trying to find the intermediate scale of the electroweak interaction of somewhere between the quark-lepton scale and the GUT or Planck scale has traditionally been approached with either a supersymmetric solution or a dynamical solution (“technicolor”). This supersymmetric solution, in some sense, is analogous to the model proposed in the appendix of Hill, Schramm and Fry[1], denoted as HSF, which is an adaptation of the Hill-Ross[29] mechanism. A dynamical solution which has been proposed by Dimopoulos[30] involves a shadow $SU3$. The scale of a physics that might be associated with an HSF mechanism was relating to the MSW mixing solution to the solar neutrino problem.

The MSW[31,32] mixing solution to the solar neutrino problem is achieved if the neutrino mass difference squared, δm^2 , is of the order of 10^{-4} to $10^{-7} eV^2$, or, in other words, neutrino masses of the order of a fraction of an electron volt. If we assume, following HSF, that the neutrino masses are generated by a pseudo-Nambu-Goldstone boson mechanism with mass

$$m_\phi \sim \frac{m_\nu^2}{f}$$

and with a transition occurring at $T_{crit} \sim m_\nu$, and if we further assume that the coupling f is related to the GUT scale, since we want to imbed this in some sort of unified theory, then the Compton wavelength $\lambda_\phi \sim 1Mpc$, in other words, a galactic scale. The density of the ϕ field at the time of the transition is the order of the cosmological density, in other words,

$$\frac{\rho_\phi}{\rho} \sim 1.$$

(Note that this is natural for phase transitions, whereas the requirement for primordial

transitions to have small fluctuations, as inflation requires, is a fine tuning requirement.) Furthermore, the average spacing of the nucleation sights, L , can be estimated from Coleman's theory on spontaneous nucleation to yield spacings today that are $\sim 100Mpc$:

$$\frac{R_H}{L} \sim \text{Log}\left(\frac{M_p}{T_{crit}}\right)$$

$$L_{co} \equiv L(1 + z_{crit}) = \frac{6000}{\text{Log}\left(\frac{M_p}{T_{crit}}\right)} \left(\frac{0.5}{h_0}\right) (1 + z_c)^{-1/2}$$

where $1 + z_{c(crit)} = \frac{T_{crit}}{T_0}$, R_H is the horizon radius at z_c and $M_p \sim 10^{19} GeV$. This yields for $T_{crit} \sim 10^{-2} eV$ to $10^{-3} eV$ $L_{co(moving)} \sim 40$ to $120Mpc$.

Recent impetus for new physics at this energy scale has come from the SAGE experiment which detects neutrinos from the PP chain in the sun. The previous solar neutrino experiments, the Chlorine and the Kamiokande experiments, are mainly sensitive to the rare 8B branch of the solar energy generating reactions. It is well established that the 8B experiments have seen fluxes at levels somewhat below theoretical predictions[33]. However, there has always been the worry that the 8B channel may be suppressed due to astrophysical effects since its yield is very temperature sensitive. However, the PP chain that produces the neutrinos to be detected by SAGE must work if the sun is burning by fusion. Thus, the report[34] of no significant counts above background after five months of running the gallium experiment when they expected nineteen counts for the standard model implies that something is happening to the neutrinos on their way between emission and arrival at earth. Of course, the present results are very preliminary. Questions with regard to estimates of background, counting efficiencies, systematics, statistics, etc., remain, but the tantalizing hint that the ν_e 's mixed into some other species of neutrino on their way out of the sun is certainly exciting. The final state of this experiment will not be known for several years. In 1991 we will begin to have results from a similar gallium experiment operated by the GALLEX collaboration in the Grand Sasso Tunnel in Italy. Their chemistry is somewhat cleaner and we will have an independent check. Furthermore, both of these gallium experiments will be calibrated using ^{51}Cr sources of MeV neutrinos. Thus, one will have a true check of their counting efficiencies, etc., and both of these experiments will run for a long-enough time that the statistics will reach significant levels. If the neutrinos really are mixing on their way out of the sun, then the MSW solution is probably valid and we are in the realm discussed above.

It might also be noted that a simple application of the Gell-Mann-Ramond-Slansky see-saw model[35] for neutrino masses yields some interesting implications. If we assume that there is a mass hierarchy in the neutrinos with the electron neutrino having negligible mass, the μ the intermediate mass and the τ the heaviest, and we assume that the mixing of the ν_e in the sun goes to its nearest neighbor family, the ν_μ , then the ν_μ is carrying most of the mass of the MSW δ_m^2 . The see-saw mechanism argues that

$$m_{\nu_i} \sim \frac{m_{f_i}^2}{M}$$

for a given family, or, in other words,

$$m_{\nu_\tau} \sim m_{\nu_\mu} \left(\frac{m_{f_3}}{m_{f_2}}\right)^2.$$

If we use lepton masses for the fermion masses, this yields a ν_τ mass in the neighborhood of a few eV . However, if we use heavy quark masses, then, since the top quark mass is $\gtrsim 100$ times that of the charm quark, this yields ν_τ masses in the neighborhood of 10 to 100 eV , making it perfect hot dark matter. It might also be noted that the see-saw mass scale, M , in this picture, ends up being the order of 10^9 to $10^{12} GeV$, which happens to be the only window allowed for the DFS-axion[36] scale. It might further be noted that if the non-baryonic dark matter is indeed the τ neutrino, then one is required to dismiss primordial gaussian fluctuations.

Note that even if the MSW mixing is $\nu_e - \nu_\tau$, the LTPT possibility is still there, but then all neutrinos would be light and could not serve as HDM. It is interesting that in this latter case the see-saw M is the GUT scale.

Structure from LTPT

LTPT can produce vacuum fluctuations of the random gaussian character just as could be generated at the end of inflation[11], however, as emphasized in Ref. [15,16], these structures will have a quantum scale that is the order of a galaxy size, and the bosons associated with the fluctuations might even serve as the dark matter of the universe.

The other alternative for LTPT is to produce topological structures. Just as early universe phase transitions can produce strings and/or textures, LTPT can also produce such objects. Furthermore, LTPT can produce walls which are a problem for primordial phase transitions. However, there is a problem for some walls, depending on the nature of the interaction potential. LTPT that have a $\lambda\phi^4$ potential will end up with one wall dominating as was demonstrated in Ref. [6,7,8]. However, this problem of one wall dominating can be surmounted in a variety of ways which have varying degrees of attractiveness, depending on the eyes of the beholder. For example, in the HSF phase transition, the walls are sine-Gordon rather than $\lambda\phi^4$. As Widrow has shown[37], the sine-Gordon walls can yield "bags" of wall or "balls" of wall which survive several expansion times. These bags or balls can then serve as seeds in galaxy formation, and thus, it is their amplitude that becomes a deciding factor for $\frac{\Delta T}{T}$ limits as opposed to the energy scale of the infinite walls which can be made quite small. This latter point was emphasized by Hill, Schramm and Widrow[17]. Another way of avoiding single wall dominance is the decaying wall model of Kawano[38] where the walls serve as seeds and then decay away. It is also possible to escape one-wall domination with a large number of minima in the potential. Perhaps the most dramatic way of escaping one-wall domination, thus keeping a network of walls, as shown in Fig. 2A, is if the walls have friction with the ambient medium, whether it be neutrinos or the remaining baryonic and/or non-baryonic matter in the universe[39]. Alessandro Massarotti has shown that friction can in many reasonable cases slow the walls down sufficiently that they do not evolve to the one-ball domination situation. In this case, one retains a complex network with L for the wall being much less than the horizon size.

It might be noted that long walls gravitationally repel rather than attract[40,41], whereas balls of wall are attractive seeds, thus a combined network of balls and slowed-down long walls can yield a complex structure which may be even of a fractal character in agreement with the claims of Schramm and Szalay[25] from cluster correlations.

In addition to walls, LTPT can also produce textures[42] or non-topological solitons[43]. In these latter cases, or with the bags of wall dominating, one will have networks more closely resembling Fig. 2B and Fig. 2A. It should be noted that the parameters L and δ

WALL NETWORK

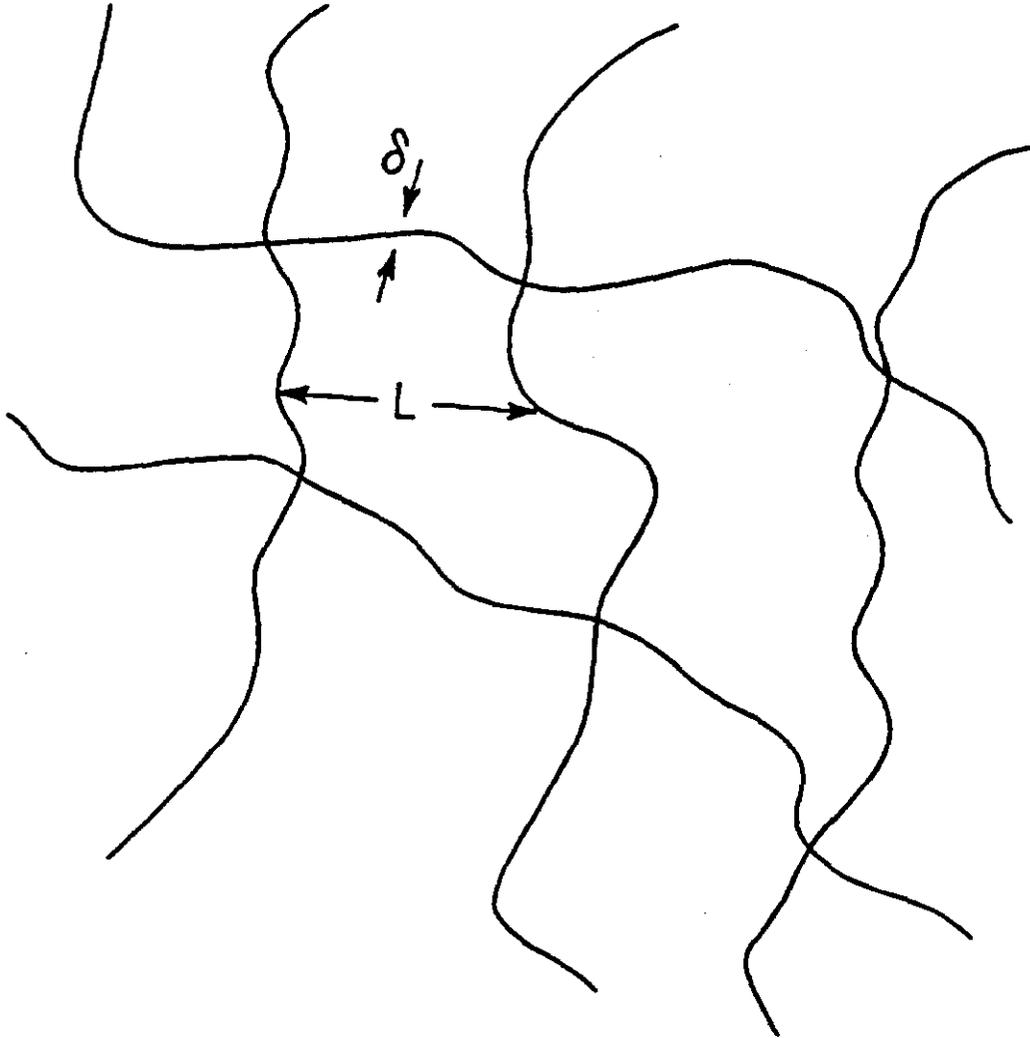


Figure 2a. A generic wall network defining the wall thickness δ and the characteristic spacing of structure L .

SEED NETWORK
(Bag, balls-of-wall, Textures, etc)

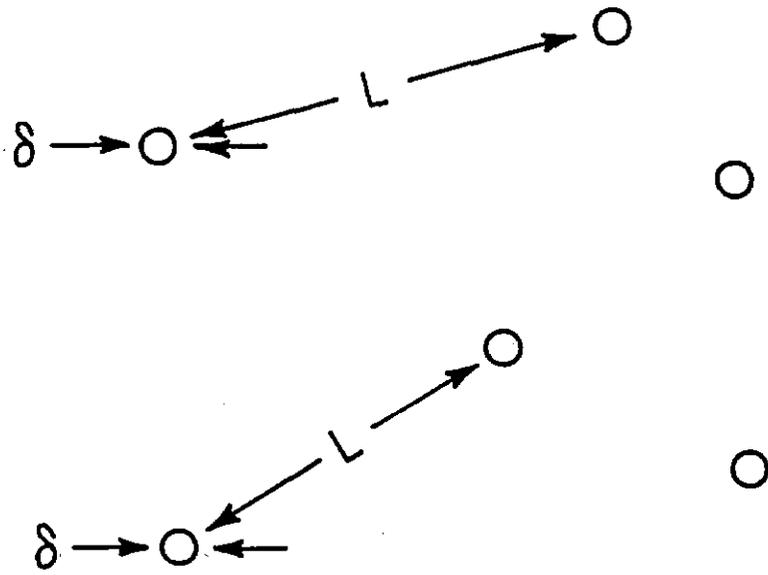


Figure 2b. A generic network for seed generation with seed size, δ and seed separation L .

and the nature of the structures generated are dependent on the model for the LTPT. It should also be noted that questions of the detailed physics of imbedding the LTPT into some larger GUT or TOE are dependent on the unification model. HSF have shown that a reasonable toy model can be constructed which can give a phase transition. These phase transitions in many ways are quite analogous to the axion-producing phase transition which has a coupling at a scale near to the order of $10^{11} GeV$, far above the QCD phase transition scale of the order of GeV . And like the axion, the particle involved in the LTPT of HSF has a pseudo-Nambu-Goldstone boson. However, instead of being related to the strong interaction and quarks, in the LTPT case it is related to the neutrinos and probably to family symmetry.

Generating seeds at an LTPT might be advantageous for producing the multiple walls of Broadhurst *et al.*[4,5]. In particular, Icke and Weygaert[2], and Coles[3] have independently demonstrated that the phenomenological Vornoi tessalations of the intersection of expanding rarefaction shells give a very good fit to large scale structure if the nodes of these tessalations are fit to the Abell clusters. In particular, they note that one gets quasi-periodic walls at $\sim 130 Mpc$ with cluster correlation functions that are quite strong and follow the fractal behavior of Schramm and Szalay. However, the seed distribution required to give this tessalation causes a conflict with the microwave background radiation, if the seeds are generated prior to the decoupling. However, an LTPT could remedy that. Similarly, an LTPT can provide the seeds to enable hot dark matter to work as a galaxy formation model (see, for example, Ref. [44]). It might be noted that the typical bag of wall can easily yield a galaxy or a quasar-forming seed.

We can estimate the mass associated with a wall in the following way:
Let $\sigma =$ energy density per unit area, that is:

$$\sigma \equiv \rho_w \delta \sim \frac{4 \times 10^{-5} s \rho_0 \delta}{h_0^2} (1 + z_c)^4$$

where

$$\delta \equiv \text{thickness}$$

$$\rho_w = \text{density of the wall}$$

$$s = \frac{\rho_w}{\rho_r} \text{ at } z_c$$

$$\rho_0 = 3 \times 10^{11} h_0^2 M_\odot / Mpc^3$$

then

$$M_w \sim \pi \sigma L^2 \sim 3 \times 10^7 (1 + z_c)^4 \left(\frac{\delta}{Mpc}\right) \left(\frac{L}{Mpc}\right)^2 M_\odot.$$

and for stable walls,

$$\Omega_w(z) \sim \frac{3}{4} \frac{\sigma}{\rho_0 L (1 + z)^3}.$$

Note that Ω_w , at the present epoch, can be the order of unity. Wall domination can occur at the present epoch if

$$z_c \gtrsim 11 \left(\frac{L}{\delta}\right)^{\frac{1}{4}} \left(\frac{h_0^2}{s}\right)^{\frac{1}{4}} - 1$$

for stable walls. It might be noted that if wall domination occurs at the present epoch, as long as there are multiple walls, rather than just one wall dominating, one has the interesting situation where the expansion of the universe is no longer following the normal matter-dominated relationship, and, in particular, one can achieve ages greater than $\frac{1}{H_0}$. Such a situation may be a solution to the age-Hubble constant problem if h_0 is ever shown to be greater than 0.7.

It might also be noted for topological structure generated by LTPT that the structure is relatively independent of whether the non-baryonic dark matter is hot or cold.

Microwave Anisotropies

Since LTPT provide no fluctuations on the surface of last scattering, all fluctuations from the microwave background must be due to the differential redshift-blueshift non-cancellation due to a changing potential in the transparent medium or due to scattering of the microwave photons off of moving objects. One can estimate the potential change due to the ϕ field itself generated in the phase transition and by the dynamic motion of the structures and the Doppler shift thereby produced. One can also do the classical Rees-Sciama and Sachs-Wolf calculations for the $\frac{\Delta T}{T}$ generated by existing objects[45,46]. We can estimate its effects roughly in the following way: The static effects will dimensionally go as

$$\frac{\delta T}{T} \sim \Omega_w \left(\frac{L}{R_H}\right)^2 \sim G\sigma L$$

The time-changing effects can be estimated by multiplying the static effect by $\frac{V}{c}$. While different people remember different formulations of these things, one can show that because of the nature of walls and other topological systems, the effects can be reduced to the form $G\sigma L$ times $\frac{V}{c}$ or $\frac{V^2}{c^2}$. Since any walls or topological seeds we ever see must be moving with $V < c$, the dominant effect will in general go like $G\sigma L$, which can be shown to yield the result:

$$\frac{\delta T}{T} \sim 10^{-8} \left(\frac{1+z_c}{10}\right)^4 s \left(\frac{\delta}{Mpc}\right) \left(\frac{L}{Mpc}\right)$$

$$\sim 10^{-6} \text{ for } L \sim 100Mpc, \delta \sim 1Mpc, z_c \sim 10.$$

Note that this yields $\frac{\delta T}{T} \sim 10^{-6}$ even for an L of $100Mpc$. The distribution, however, of these fluctuations depends very much on the detailed topological nature of the structures produced. In particular, Turner, Watkins and Widrow[46] have shown that balls of wall tend to produce spikes very similar in nature to the spikes that textures produce. A general formalism showing the wide range of structures in non-gaussian microwave background fluctuations has been developed by Goetz and Noetzold[47].

In general, one can see that if structure of size L is generated by a late-time phase transition, and L is the maximum size of structure produced in that transition, then the late-time transition does give the minimum $\frac{\delta T}{T}$ for that structure. Of course the question is

what is the characteristic size L of structure generated in a transition. For $\lambda\phi^4$ structures, L goes to the horizon size, in which case $\frac{\delta T}{T}$ gets larger than current observational limits. However, as mentioned above, many other possibilities can be generated in LTPT. With L at present being a somewhat freely adjustable parameter, depending on the model, the amount of friction, the decay of the walls, etc.

Future

Obviously, one of the key things that is needed for the future of LTPT is the development of more realistic particle models. While there are hints from the solar neutrino problem that interesting new flavor physics is involved at the low energy scale, truly consistent models embedded within a complete general framework remain to be worked out. Another important aspect of many of the topological models is the development of dynamical calculations with matter involved. LTPT take place when the universe is matter dominated. The evolution of the structure interacting with the matter, including friction effects, as pointed out by Massarotti, is critical. These dynamical calculations become quite complex in the same manner that the evolution of cosmic string networks became quite complex. As we've seen with cosmic string networks, it's taken many years for convergence to occur on the results. Similar complications may occur in the complex topological models generated in late-time phase transitions. This is an area that needs exploration.

Once one begins to have models for the structures generated in LTPT, one needs to generate full N-body calculations analogous to those that were carried out with cold dark matter, so that one can see whether or not realistic galaxies and structures that look like observations really do occur. The complexity of the topological structures, particularly since long walls can repel and balls of wall attract, could be quite interesting. Can one indeed generate Voronoi tessellation patterns? This remains to be seen.

Of course, from the observational point of view, the two critical things are the microwave background and the large-scale structure. In particular, if the microwave structures are indeed shown to be significantly less than 10^{-5} , whereby traditional primordial structure generating models are ruled out, then LTPT becomes attractive. Furthermore, since many LTPT involve distributions of fluctuations that are non-gaussian[47,48,49], finding such non-gaussian patterns becomes extremely important. Of course on the observational astronomy side, the actual determination of the three-dimensional large scale structure of the universe is the bottom line. Is this structure fractal in nature? Does it have patterns implying topological seeds? Can it be generated by the gravitational evolution of initial random gaussian fluctuation?

Acknowledgements

I would like to thank my collaborator on LTPT, Chris Hill, for inspiration on all aspects of this problem. I would also like to acknowledge useful discussions with my colleagues working on this exciting LTPT problem: Savas Dimopoulos, Jim Fry, George Fuller, Günter Götze, Dirk Nötzold, Bill Press, Graham Ross, Barbara Ryden, David Spergel, Albert Stebbins, Glen Starkman, Michael Turner, Rick Watkins, Larry Widrow. This work was supported in part by NSF grant AST 88-22595 and by NASA grant NAGW 1321 at the University of Chicago and by the DoE and NASA grant NAGW 1340 at the NASA/Fermilab Astrophysics Center.

References

- [1] Hill, C., Schramm, D.N., and Fry, J.N. (1990) "Cosmological structure formation from soft topological defects," *Comments Nuclear and Particle Physics*, 19, 25-39.
- [2] Icke, V. and van de Weygaert, R. (1990) "Vornoi Cosmology," in press, *Quarterly Journal of the Royal Astronomical Society*.
- [3] Coles, P. (1990) "Understanding recent observations of large-scale structure," submitted to *Mon. Notices Royal Astronomical Society*.
- [4] Broadhurst, T., Ellis, R., Koo, D., and Szalay, A. (1990) "Large-scale distribution of galaxies at the Galactic poles," *Nature*, 343, 726.
- [5] Koo, D.C. and Kron, R.G. (1988) "A deep redshift survey of field galaxies," in F.G. Kron and A. Renzini (eds.), *Towards Understanding Galaxies at Large Redshifts*, Kluwer Academic Publishers, Dordrecht, p.209.
- [6] Press, W., Ryden, B., and Spergel, D. (1989) "Dynamical evolution of domain walls in an expanding universe," *Astrophysical Journal*, 397, 590.
- [7] Hodges, H. (1988?) "Domain walls with bound Bose condensates," *Physical Review D*, 37, 3052-5.
- [8] Stebbins, A.J. and Turner, M.S. (1989) "Is the great attractor really a great wall?" *Astrophysical Journal*, 339, L13.
- [9] Smoot, G. (1990) "The Microwave background," *Proceedings of the Tokyo meeting on Primordial Nucleosynthesis*, September 1990.
- [10] Schneider, D.P., Schmidt, M., and Gunn, J.E. (1989) "A Quasar at $z = 4.73$," *Astron. Journal*, 98, 1951-1958.
- [11] Efstathiou, G. and Rees, M.J. (1988) "High red-shift quasars in the Cold Dark Matter cosmogony," *Mon. Notices Royal Astronomical Society*, 230, 5-11.
- [12] Olive, K. (1990) "Inflation," *Physics Reports*, 190, 309-403.
- [13] Kolb, E. and Turner, M. (1989) *The Early Universe*, Addison Wesley, San Francisco.
- [14] Schramm, D.N. (1990) "Big bang nucleosynthesis: the standard model and alternatives," *Proc. of the 1990 Nobel Symposium*, Gra ft valen, Sweden.
- [15] Wasserman, I. (1986) "Late phase transitions and spontaneous generation of cosmological density perturbations," *Physics Review Letters*, 57, 2234.
- [16] Press, W., Ryden, B., and Spergel, D. (1990) "Single mechanism for generating large-scale structure and providing dark missing matter," *Physical Review Letters*, 64, 1084.
- [17] Hill, C.T., Schramm, D.N., Widrow, L.M. (1990) "Late-time phase transitions and large scale structure," *Proc. of the XXVth Rencontres de Moriond at Les Arcs, Savoie, France, March 1990*, in press, and *Science*, submitted.
- [18] Frieman, J., Hill, C. and Watkins, R. (1990) "Late-time phase transitions," *Fermilab Preprint*, in press.
- [19] Geller, M. and Huchra J. (1989) "Mapping the universe," *Science*, 246, 897.

- [20] Szalay, A. (1990) "Correlations of galaxies over cosmic scales," Proc. IUPAP Conf. Primordial Nucleosynthesis and Evolution of Early Universe, Tokyo, September 1990.
- [21] Faber, S. and Dressler, A. (1988) "The Great attractor," Proc. Texas Symposium on Rel. Astrophysics, Dallas.
- [22] Bahcall, N. and Soneira, R. (1983) "The Spatial correlation function of rich clusters of galaxies," *Astrophysical Journal*, 270, 20-38.
- [23] Klypin, A. and Kopylov, A. (1983) "Cluster correlations," *Soviet Astr. Lett.*, 9, 41-46.
- [24] Primack, J. and Dekel, A. (1990) U. of California at Santa Cruz preprint.
- [25] Szalay, A. and Schramm, D.N. (1985) "Are galaxies more strongly correlated than clusters?" *Nature*, 314, 718-719.
- [26] Huchra, J. (1990) Harvard University preprint.
- [27] van den Bergh, S. and West, J. (1990) "Correlation Functions of CD Galaxies," Dominion Astronomical Observatory preprint.
- [28] Fuller, G. and Schramm, D.N. (1990) "Neutrino flypaper and the formation of structure in the universe," University of California at San Diego preprint.
- [29] Hill, C. and Ross, G. (1988) "Pseudo-Goldstone bosons and new macroscopic forces," *Physics Letters B*, 203, 125.
- [30] Dimopoulos, S. (1990) private communication.
- [31] Mikheyev, S.P. and Smirnov, A.Yu.. (1986) "Resonant amplification of ν oscillations in matter and solar-neutrino spectroscopy," *Nuovo Cimento*, 9C, 17.
- [32] Wolfenstein, L. (1978) "Neutrino oscillations in matter," *Physical Review D*, 17, 2369.
- [33] Bahcall, J. (1989) *Neutrino Astrophysics*, Cambridge University Press, Cambridge, England and New York.
- [34] SAGE colloboration (1990) Int. High energy Meeting, Singapore, 1990.
- [35] Gell-Mann, M, Ramond, P. and Slansky, R., (1979) "Complex spinors in unified theories," in *Supergravity*, P. Van Nieuwenhuizen and D. Freedman (eds.), North Holland Pub. Co.
- [36] Dine, M., Fischler, W., and Srednicki, M. (1981) "A Simple solution to the strong CP problem with a harmless axion," *Physics Letters B*, 104, 199.
- [37] Widrow, L. (1990) "Wall interactions," CfA/Harvard preprint.
- [38] Kawano, L. (1990) "Evolution of domain walls in the early Universe," *Physical Review D*, 41, 1013.
- [39] Massarotti, A. (1990) "Evolution of light domain walls interacting with dark matter," Fermilab Pub-90/77-A, and *Physical Review D*, in press, 1990.
- [40] Ipser, J. and Sikivie, P. (1984) "Gravitationally repulsive domain walls," *Physical Review D*, 30, 712.
- [41] Götz, G. (1990) "Scalar field configurations with planar and cylindrical symmetry: Thick domain walls and strings," Fermilab PUB-90/35-A.

- [42] Turok, N. (1989) "Textures," *Physical Review Letters*, 63, 2625.
- [43] Lee, T.D. (1987) "Soliton stars and the critical masses of black holes," *Physical Review D*, 35, 3637.
- [44] Bertschinger, E., Scherrer, R., Vilenos, J., and VanDalen, A. (1990) "Seeded hot dark matter," Ohio State preprint.
- [45] Rees, M. and Sciama, D. (1968) "Cosmological background radiation," *Nature*, 217, 511.
- [46] Sachs, R.K. and Wolf, A.M. (1967) "Perturbations of a cosmological model and angular variations of the microwave background," *Astrophysical Journal*, 147, 74.
- [47] Turner, M., Watkins, R., and Widrow, L. (1990) "Microwave distortions from collapsing domain-wall bubbles," *Astrophysical Journal Letters*, submitted.
- [48] Götz, G. and Nötzold, D. (1989) "On thick domain walls in General Relativity," Fermilab PUB-89/235-A and (1990) *Physical Review D*, submitted; and Götz, G. and Nötzold, D. (1989) "An exact solution for a thick domain wall in general relativity," Fermilab PUB-89/236-A and (1990) *Physical Review D*, submitted.
- [49] Stebbins, A. (1990) "Microwave anisotropy signatures," CITA preprint.